Problem of the Week Problem C and Solution New Heights (Revised)

Problem

An *altitude* is a line segment drawn from a vertex of a triangle to the opposite side or opposite side extended such that the line segment is perpendicular to the opposite side. In $\triangle ABC$, CD is an altitude. AB = 18 cm, AC = 20 cm and CD = 16 cm. An altitude is drawn from B to AC intersecting at E. Determine the length of BE.



Solution

The area of a triangle is determined using the formula $base \times height \div 2$. The height of the triangle is the length of an altitude and the base of the triangle is the length of the side to which a particular altitude is drawn.

Area
$$\triangle ABC = \frac{(CD) \times (AB)}{2}$$

 $= \frac{16 \times 18}{2}$
 $= 144 \text{ cm}^2$
But, Area $\triangle ABC = \frac{(BE) \times (AC)}{2}$
 $144 = \frac{(BE) \times 20}{2}$
 $144 = 10 \times BE$
 $14.4 \text{ cm} = BE$

Therefore, the length of altitude BE is 14.4 cm.



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Problem of the Week Problem C and Solution New Heights (Original Problem)

Problem

An *altitude* is a line segment drawn from a vertex of a triangle to the opposite side or opposite side extended such that the line segment is perpendicular to the opposite side. In $\triangle ABC$, CD is an altitude. AB = 16 cm, AC = 12 cm and CD = 6 cm. An altitude is drawn from B to AC extended intersecting at E. Determine the length of BE.



Solution

The area of a triangle is determined using the formula $base \times height \div 2$. The height of the triangle is the length of an altitude and the base of the triangle is the length of the side to which a particular altitude is drawn.

Area
$$\triangle ABC = \frac{(CD) \times (AB)}{2}$$

 $= \frac{6 \times 16}{2}$
 $= 48 \text{ cm}^2$
But, Area $\triangle ABC = \frac{(BE) \times (AC)}{2}$
 $48 = \frac{(BE) \times 12}{2}$
 $48 = 6 \times BE$
 $8 \text{ cm} = BE$

Therefore, the length of altitude BE is 8 cm.

